## RELATIONSHIP BETWEEN MOISTURE CONTENT AND TEMPERATURE OF DIS-PERSED MATERIAL IN DRYING IN A FLUIDIZED BED

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Solution of the differential calorimetric equation of the heat balance of a fluidized bed leads to a relationship between the moisture content and the temperature of the dispersed material during the constant and falling drying rate periods.

One of the most topical problems in the investigation of rapid drying of various materials is the establishment of a single-valued relationship between the moisture content and the temperature of the material during its dehydration [1-6]. The solution of this problem is particularly important for the technology of drying of dispersed thermally unstable materials in the suspended state and, in particular, in a fluidized bed. The possibility of considerable acceleration of heat and mass transfer, on the one hand, and the risk of overheating the material and adversely affecting the quality of the final product, on the other, makes it essential to choose optimum process parameters and to design drying devices which allow a thermally unstable material to be heated to the maximum permissible temperature.

The literature contains mathematical descriptions of the kinetics of heating of moist dispersed material during the constant and falling drying rate periods in batch drying in a fluidized bed [5, 7, 8]. In continuing these investigations in this paper we will attempt to examine analytically the problem of the direct relationship between the moisture content and the temperature of the dispersed material in batch drying in fluidizedbed apparatuses.

It is well known that in general a rigorous analytical investigation of heat and mass transfer in heterogeneous dispersed systems encounters insuperable difficulties. However, the special characteristics of fluidized-bed apparatuses due to the vigorous mixing of the phases throughout the volume of the chamber allow several assumptions which simplify the solution of the problem. In particular, as experimental tests have shown, a perfectly satisfactory basis for the development of an approximate method of calculating the kinetics of heating of a moist dispersed material in a fluidized bed is provided by the following simplifying assumptions [5, 7].

1. At every instant the fluidized bed of material undergoing drying is assumed to be isothermal, and the temperature of the spent drying agent on emergence from the bed is equal to the integral temperature of the particles in the bed.

2. The temperature gradient over the cross section of the particles is assumed to be negligibly small [9].

3. In batch drying the moisture content of the particles throughout the volume of the fluidized bed is assumed to be the same at the same instant.

Adopting this model and neglecting heat loss to the surroundings we can put the calorimetric equation of

the heat balance of a fluidizing bed of moist dispersed material in the following form:

$$F_{\rm s} v \rho c \left(t - \Theta\right) d\tau = G c_{\rm m} d\Theta - G_{\rm d.s.} r du. \tag{1}$$

For the constant drying rate period

$$d\tau = -du/N.$$

Expressing the mass of the material and its specific heat for any instant by

$$G = G_{d.s} (1 + u),$$
  
 $c_{m} = \frac{c_{d.s} + u}{1 + u},$ 

respectively, we obtain after some transformations

$$(B\Theta - A) du = (c_{ds} + u) d\Theta, \qquad (2)$$

where

$$B = \frac{F_s v \rho c}{G_{d,s} N}; \quad A = Bt - r.$$

Separating the variables and integrating in the appropriate limits we finally obtain

$$u = (c_{d,s} + u_1) \left( \frac{A - B\Theta}{A - B\Theta_0} \right)^{\frac{1}{B}} - c_{d,s}.$$
 (3)

The relationship between the temperature of the fluidized bed and the varying moisture content of the material in the constant rate period in batch drying can accordingly be written as follows:

$$\Theta = \frac{A}{B} - \left(\frac{A}{B} - \Theta_0\right) \left(\frac{c_{d.s} + u}{c_{d.s} + u_1}\right)^B.$$
 (4)

Expressions similar to (3) and (4) can be obtained after some transformations of the drying time equation given in [5].

In the general case it can be assumed with sufficient accuracy for technical calculations that the drying rate varies linearly in the falling drying rate period [2]:

$$-\frac{du}{d\tau} = K(u-u_{\rm e}).$$
<sup>(5)</sup>

Using (5) we can write the initial differential heat balance equation for a fluidizing bed of moist dispersed material in the following form:

$$-\frac{F_{s}v\rho c(t-\Theta)}{K(u-u_{e})}du = G_{d.s}(c_{d.s}+u)d\Theta - G_{d.s}rdu.$$
 (6)

Omitting the intermediate transformations, we obtain

$$\frac{d\Theta}{du} - \Theta \frac{B_1}{(u - u_e)(u + c_{d,s})} \approx$$
$$= \frac{r(u - u_e) - B_1 t}{(u - u_e)(u + c_{d,s})}, \qquad (7)$$

where

$$B_1 = F_e v \rho c / KG_{d.s}.$$

The solution of the inhomogeneous linear first-order differential equation (7) with initial condition  $\bigotimes |_{u=u_1}' = \bigotimes_0'$  gives

$$\Theta = \Theta'_{0} \left[ \frac{(u'_{1} + c_{d,s})(u - u_{e})}{(u'_{1} - u_{e})(u + c_{d,s})} \right]^{m} + \sum_{n=1}^{m-1} \left\{ \frac{r}{n - m} \left[ \left( \frac{u - u_{e}}{u + c_{d,s}} \right)^{n} - \left( \frac{u - u_{e}}{u + c_{d,s}} \right)^{m} \left( \frac{u'_{1} + c_{d,s}}{u'_{1} - u_{e}} \right)^{m-n} + \frac{B_{1}t}{m} \left[ \frac{(u - u_{e})^{n-1}}{(u + c_{d,s})^{n}} - \left( \frac{u - u_{e}}{u + c_{d,s}} \right)^{m} \frac{(u'_{1} + c_{d,s})^{m-n}}{(u'_{1} - u_{e})^{m-n+1}} \right] \right\},$$
(8)

where

$$u = \frac{B_1}{c_{d,s} + u_e}$$

n

(if m is fractional it must be rounded off to the nearest whole number).

An analysis of the obtained solution showed that for practical calculations we can obtain a satisfactory result by taking only the first term in the infinite sum contained in expression (8). Hence, the general approximate solution can be put in the following form, which is suitable for practical application:

$$\Theta = \left[\Theta'_{0} - \frac{r(u'_{1} - u_{e})}{(1 - m)(u'_{1} + c_{d,s})} - \frac{B_{1}t}{m(u'_{1} + c_{d,s})}\right] \times \left[\frac{(u - u_{e})(u'_{1} + c_{d,s})}{(u + c_{d,s})(u'_{1} - u_{e})}\right]^{m} + \frac{r(u - u_{e})}{(1 - m)(u + c_{d,s})} + \frac{B_{1}t}{m(u + c_{d,s})}.$$
(9)

Equation (9) shows that when the moisture content of the product attains its equilibrium value  $(u = u_e)$  the integral temperature of the fluidized bed of dispersed material becomes equal to the temperature of the heat transfer agent ( $\Theta = t$ ).

In this case, when the constant drying rate period precedes the falling rate period, the value of  $\Theta'_0$  can be calculated from Eq. (4).

We note that the quantity m, which is contained in expression (8), depends on the main factors affecting the kinetics of heat and mass transfer between the material and the drying agent. Hence, we must first assign a tem-

perature, mass velocity, and physical properties to the drying agent, a specific load of material on the screen, and the initial moisture content and main properties of the material undergoing drying. As our investigations showed, the actual values of parameter m for fluidized-bed drying of such thermally unstable materials as grain and shredded potato are in the range m = 10-200.



Fig. 1. Relationship @= f(u) for drying of wheat grain in a fluidized bed ( $v\rho =$ = 1.98 kg/m<sup>2</sup> · sec, G<sub>d.S</sub>/F<sub>S</sub> = 65 kg/ /m<sup>2</sup>, u<sub>1</sub> = 0.3 kg/kg, t = 80° C): 1) from Eq. (9); 2) from experimental data.

It follows from expression (8) that the mathematical error due to simplification of the obtained solution depends mainly on the value of the simplex  $(u - u_e)//(u + c_{d.s})$ . With reduction in the moisture content u the value of simplex (and, hence, the magnitude of the error introduced into the calculation) is also reduced. This must be taken into account in the estimation of the over-all error of the method, since in the determination of the relationship  $\Theta = f(u)$  the most interesting region is that of low moisture contents, when the temperature of the bed comes closest to the maximum permissible (based on technological considerations) temperature of the product.

Finally, deviation of the calculated values of the temperature of the moist material from the actual values depends largely on the nature of the drying process and on how accurately the curve of drying rate in the falling rate period is given by Eq. (5). An analysis of this error was made in [7].

The applicability of the proposed approximate solution is confirmed by a comparison of the results of calculation from Eq. (9) with experimental data obtained in an investigation of grain drying in a wide range of variation of the process parameters. The results of this comparison indicate a perfectly satisfactory agreement between the experimental and calculated data. The deviation of the experimental points from the curves calculated from the above formula did not exceed 12% (see Fig. 1).

In conclusion we note that the obtained relationships can be used to select efficient regimes of drying of dispersed thermally unstable materials in fluidized-bed apparatuses with due regard to the maximum permissible temperature of the product and to design an automatic-control system for such driers. The analytically established direct relationship between the moisture content and the temperature of the product can provide a basis for a program of automatic control of drying based on the change in the mean temperature of the fluidized bed of material. The initial parameter in this case could be the temperature of the spent drying agent, the measurement of which presents no practical difficulties and can be effected by a thermometric gauge of any type.

## NOTATION

 $F_S$  is the area of gas-distributing screen;  $v\rho$  is the mass velocity of the drying agent; c and  $c_m$  are the specific heats of the gas and the material, respectively; t is the initial temperature of the drying agent;  $\Theta$  is the temperature of material;  $\tau$  is time;  $c_{d,S}$  is the specific heat of the dry substance;  $N = (du/d\tau)_n$  is the drying rate in constant rate period; K is the drying coefficient;  $u_1$ ,  $u'_1$ ,  $u_e$  are the initial, reduced, and equilibrium moisture contents of the material, respectively;  $\Theta'_0$  is the temperature of the product with moisture content  $u'_1$ ; r is the heat of vaporization.

## REFERENCES

1. A. V. Luikov, collection: Heat and Mass Transfer, Vol. 4 [in Russian], Izd. Nauka i tekhnika, Minsk, 1966. 2. A. V. Luikov, Heat and Mass Transfer in Drying Processes [in Russian], Gosenergoizdat, 1956.

3. S.S. Zabrodskii, Hydrodynamics and Heat Transfer in a Fluidized Bed [in Russian], Gosenergoizdat, 1963.

4. G. D. Rabinovich, IFZh [Journal of Engineering Physics], 11, no. 2, 1966.

5. A.S. Ginzburg and V.A. Rezchikov, Drying of Food Products in a Fluidized Bed [in Russian], Izd. Pishchevaya promyshlennost, 1966.

6. G.K. Filonenko and P.D. Lebedev, Drying Plant (in Russian), Gosenergoizdat, 1952.

7. I. F. Pikus, collection: Heat and Mass Transfer in Dispersed Systems [in Russian], Izd. Nauka i tekhnika, Minsk, 1965.

8. I.F. Pikus, Abstracts of Papers and Communications of the All-Union Conference on Intensification of Drying Processes in the Chemical Industry [in Russian], VSNTO, Moscow, 1965.

9. I. T. El'perin and I. F. Pikus, IFZh [Journal of Engineering Physics], 12, no. 1, 1967.

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